A Nonperturbative Approach to the Problem $\pi^0 \to 2\gamma$

P. Mukhopadhyay

Department of Physics, Jadavpur University, Calcutta 32, India

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In this paper the question of the asymptotic conformal invariance of the off-shell amplitude for the process $\pi^0 \to 2\gamma$ has been investigated. It has been shown that the off-shell amplitude for the process concerned can be given an asymptotically conformal covariant form by using a parton model description of the neutral pion instead of treating the pion as an elementary particle. This investigation shows that the conclusion regarding the question of the asymptotic conformal invariance of the off-shell amplitude for the process $\pi^0 \to 2\gamma$ obtained from the parton model along with the use of conformal covariant operator product expansions is the same as that of the Johnson-Baker-Willey version of finite QED from which the asymptotic conformal invariance of the 3-point Green's function for the process $\pi^0 \to 2\gamma$ follows as a necessary consequence.

Introduction

The theoretical study of the process $\pi^0 \to 2\gamma$ has always remained a point of attraction for the intricacies involved in its theoretical treatment. The process concerned has been investigated by different workers [1–5] employing various approaches (both perturbative and nonperturbative). Apart from the fact that the study of this process can yield valuable information regarding the PCAC anomaly [2, 4], it is also especially interesting from the standpoint of conformal symmetry. It is interesting to note that according to Migdal [6] the 3-point Green's function for the process $\pi^0 \to 2\gamma$ is conformally non-covariant. Needless to mention that Migdal's treatment [6] suffers from the insufficient physical input as he has not considered the pion PCAC or the compositeness of the pion. It is to be noted that the amplitude for the process $\pi^0 \to 2\gamma$ vanishes in the naive quark model due to a well known trace theorem and the pion PCAC has to be invoked so that the amplitude survives. The use of the pion PCAC necessitates the use of the conformal Ward identity for a conformal covariant description of the process $\pi^0 \to 2\gamma$.

In this paper we have made an attempt to show the asymptotic conformal invariance of the off-shell amplitude for the process $\pi^0 \to 2\gamma$ by exploiting the parton model idea of Feynman [7] for the compositeness of the hadrons (in our case the neutral pion) instead of the usual procedure of considering the pion PCAC along with the Ward identity. The idea of a hadron as a composite system of partons was used by Bjorken and Paschos [8] in their explanations for the scaling behaviour of structure functions. The number of partons (i. e. the constituent particles) forming a composite fundamental particle is, however, not unique. It has been shown [9] that the scaling of the on-shell and the dipole power behaviour of the off-shell form factor of a neutral scalar hadron, considered as a composite system, is reproduced with two constituent particles (partons). In this investigation the neutral pion has been considered, for the sake of simplicity, as a composite system of two non-interacting spin-zero chargeless partons of equal mass. With this parton description of the neutral pion and no additional hypothesis such as the pion PCAC, an asymptotically conformal invariant off-shell amplitude for the process $\pi^0 \to 2\gamma$ has been derived in Sec. 1 using conformal covariant operator product expansions.

It is gratifying that the motivation of this paper receives a strong support from the Johnson-Baker-Willey version [10, 11] of finite QED from which the asymptotic conformal invariance of the off-shell amplitude for the process $\pi^0 \to 2\gamma$ follows as a necessary consequence.

Reprint requests to P. Mukhopadhyay, Department of Physics, Jadavpur University, *Calcutta 32*, India.

1. Formulation

It is well known that the co-ordinate transformations, which leave the metric of the Minkowski space invariant, form the conformal group whose universal covering group is the spinor group SU(2,2). The special conformal and scale transformations form with the Poincaré transformations the 15-dimensional conformal group [12]. The finite special conformal transformations, given by

$$x_\mu' = (x_\mu - a_\mu x^2)/(1 - 2a\cdot x + a^2 x^2) \quad (1)$$

are capable of converting space-like to time-like vectors and hence can violate causality. Therefore, the full conformal group, which includes the special conformal group as a subgroup, does not preserve causality. However, the infinitesimal special conformal transformations given by

$$\delta x_\mu = 2(a\cdot x)x_\mu - a_\mu x^2, \quad a_\mu \to 0 \quad (2)$$

are causal and hence the conformal group is locally causal. The conformal invariance can be used to determine uniquely the two-point and three-point functions [4]. In this paper we shall be interested in the question of covariance of the off-shell amplitude for the process $\pi^0 \to 2\gamma$ under the (infinitesimal) conformal transformations.

The amplitude for the process $\pi^0 \to 2\gamma$ is given by

$$M = \mathcal{E}^\mu \mathcal{E}^\nu W_{\mu\nu} \quad (3)$$

where $\mathcal{E}^\mu$ and $\mathcal{E}^\nu$ are the polarization four-vectors of the two photons and $W_{\mu\nu}$ can be written as (apart from the coupling constant and the well known kinematical factors)

$$W_{\mu\nu} = i \int d^4x\, d^4y \exp\{i k_1\cdot x\} \exp\{i k_2\cdot y\} (\Box + m^2) \cdot \langle 0 | T[J_\mu(x) J_\nu(y) \Phi(0)] | 0\rangle \quad (4)$$

where k_1 and k_2 are the four-momenta of the two photons and $P = k_1 + k_2$ is the pion four-momentum; Φ is the local interpolating field of the pion. It is interesting to note that the Lorentz covariant (and hence form invariant) 3-point function $\langle 0 | T[J_\mu(x) J_\nu(y) \Phi(0)] | 0\rangle$ is not conformally covariant. The non-covariance of the 3-point function $\langle 0 | T[J_\mu(x) J_\nu(y) \Phi(0)] | 0\rangle$ under infinitesimal special conformal transformations of coordinates given by Eqn. (2) follows from the transformation properties [6] of $J_\mu(x)$ and $\Phi(x)$ which are

$$\delta J_\mu(x) = 6(a\cdot x) J_\mu(x) + 2(a^\mu x^\nu - a^\nu x^\mu) J_\nu(x)$$

and

$$\delta\Phi(x) = 2\,d(a\cdot x)\Phi(x)\,.$$

As the 3-point function under consideration is not covariant under infinitesimal special conformal transformations, it is obviously non-covariant under the full conformal group of transformations.

As the Lorentz group is a subgroup of the full conformal group and since $W_{\mu\nu}$ is covariant under the Lorentz group, therefore, $W_{\mu\nu}$ cannot be made covariant under the bigger group, namely the conformal group, without the introduction of an additional physical idea. The usual procedure for giving $W_{\mu\nu}$ a conformally covariant form is to make use of the pion PCAC along with the Ward identity. Instead of following this standard procedure, we shall follow a different method. As has already been remarked in the introduction, the non-covariance [6] of the 3-point function (and hence of the amplitude) lies in treating the pion as an elementary particle or equivalently in treating the pion interpolating field Φ as an elementary field. Instead of doing this, we shall treat the pion as a bound system of two non-interacting partons. We shall assume, for the sake of simplicity, that the two constituent partons are of the same mass and zero spin and charge.

It is interesting to note that γ_5-description (i.e. pseudoscalar description) for the pion is a must when one considers the question of (asymptotic) conformal invariance of $\pi^0\gamma\gamma$ vertex in the light of the quark model as the neutral pion is a member of the pseudoscalar nonet. However it is important to remember that in the 15-dimensional (continuous) conformal group there is no room for involutive transformations such as space-inversions. Therefore, the conformal group is, truly speaking, unable to distinguish between a scalar and a pseudoscalar. Treating the pion interpolating field $\Phi(x)$ as a composite scalar field defined by [13]

$$\Phi(x) = \lim_{\xi\to 0} N\, \Phi_a(x+\xi)\, \Phi_b(x-\xi)$$

where N is the normalization constant [13], Eqn. (4) can be rewritten in the limit of scale invariance (when $\Box + m^2$ can be replaced by $\Box$) as follows

$$W_{\mu\nu} \cong i \lim_{\xi\to 0} N \int d^4x\, d^4y \exp\{i k_1\cdot x\} \exp\{i k_2\cdot y\} \Box \times \langle 0 | T[J_\mu^a(x) J_\nu^b(y) \Phi_a(\xi) \Phi_b(-\xi)] | 0\rangle\,. \quad (5)$$

The superscript a on $J_\mu^a(x)$ indicates that the current $J_\mu^a(x)$ is coupled (via an infinite set of hermitian local operators which are annihilated by the infinitesimal special conformal generator K_λ) to the field $\Phi_a(\xi)$ of the parton labelled by a. A similar remark holds for the superscript on $J_\nu^b(y)$. In coupling $J_\mu^a(x)$ to $\Phi_a(\xi)$ and $J_\nu^b(y)$ to $\Phi_b(-\xi)$, we have made explicit use of the parton model concept. We can imagine this as if the pion is being probed by two (highly virtual) photons, each interacting with only one parton via an infinite set of local operators. Now, taking advantage of the fact that $\Phi_a(\xi)$ is coupled to $J_\mu^a(x)$ only, we can replace

$$T[J_\mu^a(x) J_\nu^b(y) \Phi_a(\xi) \Phi_b(-\xi)] \text{ by}$$
$$T\{[J_\mu^a(x) \Phi_a(\xi)][J_\nu^b(y) \Phi_b(-\xi)]\}.$$

It may be argued that the replacement just mentioned is not justified because of the singularities in the short distance or the light-cone behaviour of the operator products. It is important to remember that the consideration of the singularities of the operator products appearing in the expression for an amplitude is meaningful if the amplitude itself is covariant and hence physical. It is to be noted that $W_{\mu\nu}$ given by Eq. (4) does not become conformally covariant by the replacement of the elementary pion interpolating field by the composite field. The expression for $W_{\mu\nu}$ given by expression (5) is still conformally non-covariant and hence unphysical from the standpoint of conformal symmetry. Therefore, the replacement mentioned above can only affect the singularity structures of the unphysical (i. e. non-covariant) amplitude. Hence, by making the replacement indicated above and taking the limit $\xi \to 0$, the expression (5) can be given the following form

$$W_{\mu\nu} = i N \int d^4x\, d^4y \exp\{i k_1 \cdot x\} \exp\{i k_2 \cdot y\} \Box \times \langle 0 | T\{[J_\mu^a(x) \Phi_a(0)][J_\nu^b(y) \Phi_b(0)]\} | 0 \rangle \quad (6)$$

which is conformally covariant as both $J_\mu^a(x) \Phi_a(0)$ and $J_\nu^b(y) \Phi_b(0)$ can be expressed in terms of conformally covariant quantities [12].

The conformally covariant operator product expansion [14] for the light-like separations is given by

$$J_\mu(x) \Phi(0) \underset{x^2 \to 0}{\sim} (\partial_\mu \partial^\varrho - g_\mu{}^\varrho \Box) \sum_n \left(\frac{1}{-x^2 + i\varepsilon} \right)^{\frac{1}{2}(l - \tau_n)} \times x^{\alpha_1} \ldots x^{\alpha_{n-1}} \int_0^1 du\, f_n(u)\, O^n_{\varrho \alpha_1 \ldots \alpha_{n-1}}(u x) \quad (7)$$

where
$$f_n(u) = c_n u^{(d_n - l)/2} (1 - u)^{(d_n - l^*)/2} \quad (7\,a)$$
and
$$\tau_n = l_n - n, \quad d_n = l_n + n, \quad l^* = 4 - l \quad (7\,b)$$

where τ_n is the twist and l is the physical dimension. The operator $(\partial_\mu \partial^\varrho - g_\mu{}^\varrho \Box)$ takes care of the current conservation, and $O^n_{\alpha_1 \ldots \alpha_n}$ are a set of local tensor operators of given dimension l_n and irreducible under the conformal algebra i. e.

$$[O^n_{\alpha_1 \ldots \alpha_n}(0), K_\lambda] = 0$$

where K_λ is the generator of infinitesimal special conformal transformations.

Now, conformal invariance implies [12] the absence of the $n = 0$ representation in the case of $J_\mu(x) \Phi(0)$ where $J_\mu(x)$ is a conserved current. For the sake of simplicity, we shall consider the case $n = 1$. In this case we have, using the expansions of the form given by the expression (7),

$$\begin{aligned} &\langle 0 | T\{[J_\mu^a(x) \Phi_a(0)][J_\nu^b(y) \Phi^b(0)]\} | 0 \rangle \\ &\sim (\partial_\mu \partial^\varrho - g_\mu{}^\varrho \Box)(\partial_\nu \partial^\sigma - g_\nu{}^\sigma \Box) \\ &\cdot (x^2)^{-(l - \tau_1)/2} (y^2)^{-(l - \tau_1)/2} \\ &\times \int\int du\, du'\, f_1(u) f_1(u') \langle 0 | O_\varrho(u x) O_\sigma(u' y) | 0 \rangle . \end{aligned} \quad (8)$$

Under the most general conformal transformations (including the involutive transformation such as conformal inversion) the covariant 2-point Wightman function for a vector field of dimension l_1 is given by [15]

$$\langle 0 | O_\varrho(x_1) O_\sigma(x_2) | 0 \rangle = \frac{\Gamma(l_1)}{4\pi^2} \cdot \left(\frac{g^{\varrho\sigma}}{2 - l_1} \Box + \frac{2}{l_1 - 1} \partial^\varrho \partial^\sigma \right) \left(\frac{4}{x_{12}{}^2} \right)^{(l_1 - 1)} \quad (9)$$

where

$$x_{12} = [i\, 0 (x_1{}^0 - x_2{}^0) - (x_1 - x_2)^2]^{1/2}$$

and $l_1 = 3$ for a conserved vector field only.

Interpreting the local operators $O_\varrho(u x)$ and $O_\sigma(u' y)$ appearing in the expression (8) as the vector field operators and using the Eq. (9) and also the generalized Feynman integral

$$(x_1^2)^{-a} [(x_1 - x_2)^2]^{-b} = \frac{\Gamma(a + b)}{\Gamma(a)\Gamma(b)} \int_0^1 d\lambda\, \lambda^{b-1} (1 - \lambda)^{a-1} [(x_1 - \lambda x_2)^2]^{-(a+b)}$$

along with the Fourier transform

$$(x^2)^{-d} = \int e^{-ik \cdot x} (k^2)^{d-2} d^4k$$

the expression (8) can be rewritten in the following form

$$\langle 0 | T\{[J_\mu^a(x) \Phi_a(0)][J_\nu^b(y) \Phi_b(0)]\} | 0 \rangle \sim \frac{i N \Gamma(l_1) (2)^{(2 l_1 - 2)}}{4\pi^2} \left\{ \frac{\Gamma\left(\frac{l - \tau_1 + l_1 - 1}{2}\right)}{\Gamma\left(\frac{l - \tau_1}{2}\right) \Gamma\left(\frac{l_1 - 1}{2}\right)} \right\}^2$$

$$\times \int \ldots \int d\lambda\, d\lambda'\, du\, du'\; d^4k\, d^4k'\, (u^2)^{(l-\tau_1)/2}\, (u'^2)^{(l-\tau_1)/2}\, (\lambda)^{(l_1-3)/2}$$
$$\times (1-\lambda)^{(l-\tau_1-2)/2}\, (\lambda')^{(l_1-3)/2} (1-\lambda')^{(l-\tau_1-2)/2} \tag{10}$$
$$\times \exp\{-i k\cdot (u x - \lambda u' y)\}\, \exp\{-i k'\cdot (u' y - \lambda' u x)\}\, (k^2)^{\left(\frac{l-\tau_1+l_1-1}{2}-2\right)} (k'^2)^{\left(\frac{l-\tau_1+l_1-1}{2}-2\right)}.$$

With the help of the expression (10), Eq. (6) can be reduced to the following form (after doing the partial integration and letting the surface term go to zero)

$$W_{\mu\nu} \sim \frac{i N \Gamma(l_1)\, (2)^{(2l_1-2)}}{4\pi^2} \left\{ \frac{\Gamma\left(\frac{l-\tau_1+l_1-1}{2}\right)}{\Gamma\left(\frac{l-\tau_1}{2}\right)\Gamma\left(\frac{l_1-1}{2}\right)} \right\}^2 \{-(k_1+k_2)^2\}$$
$$\times \int \ldots \int d\lambda\, d\lambda'\, du\, du'\; d^4k\, d^4k'\, (u^2)^{(l-\tau_1)/2}\, (u'^2)^{(l-\tau_1)/2}\, f_1(u)\, f_1(u') \tag{11}$$
$$\times (\lambda)^{(l_1-3)/2}\, (1-\lambda)^{(l-\tau_1-2)/2}\, (\lambda')^{(l_1-3)/2}\, (1-\lambda')^{(l-\tau_1-2)/2}$$
$$\times \{k_1^{\mu}(\lambda k - k')^{\varrho} - g_{\mu}{}^{\varrho}\, k_1^2\}\, \{k_2^{\nu}(k-\lambda' k')^{\sigma} - g_{\nu}{}^{\sigma}\, k_2^2\} \left\{ -\frac{g^{\varrho\sigma}}{2-l_1}(k-k')^2 + \frac{1}{(l_1-1)} k^{\varrho} k'^{\sigma} \right\}$$
$$\times (k^2)^{\frac{l-\tau_1+l_1-1}{2}-2}\, (k'^2)^{\frac{l-\tau_1+l_1-1}{2}-2}$$
$$\times \int d^4x\, \exp\{i(k_1 - u k + \lambda' u k')\cdot x\} \int d^4y\, \exp\{i(k_2 - u' k' + \lambda u' k)\cdot y\}.$$

The k and k' integrations are made easy by the δ-functions manufactured by x and y integrations. We get after some straightforward calculations

$$W_{\mu\nu} = C\, \frac{i N \Gamma(l_1)\, (2)^{(2l_1-2)}}{4\pi^2} \left\{ \frac{\Gamma\left(\frac{l-\tau_1+l_1-1}{2}\right)}{\Gamma\left(\frac{l-\tau_1}{2}\right)\Gamma\left(\frac{l_1-1}{2}\right)} \right\} e_{\mu\nu\alpha\beta}\, k_1^{\alpha}\, k_2^{\beta}\, (k_1+k_2)^2$$
$$\times \int \ldots \int d\lambda\, d\lambda'\, du\, du'\; f_1(u)\, f_1(u')\, (u^2)^{(l-\tau_1)/2}\, (u'^2)^{(l-\tau_1)/2}$$
$$\times (\lambda)^{(l_1-3)/2}\, (1-\lambda)^{(l-\tau_1-2)/2}\, (\lambda')^{(l_1-3)/2}\, (1-\lambda')^{(l-\tau_1-2)/2} \tag{12}$$
$$\times \left\{ \frac{(k_1\cdot k_2)}{u u' (l_1-2)} \left(\frac{k_1}{u} - \frac{k_2}{u'}\right)^2 + k_1^2 k_2^2\, \frac{(1-u)(1-u')}{u^2 u'^2 (l_1-1)} \right\}$$
$$\times \left\{\left(k_1 + \lambda' \frac{u}{u'} k_2\right)^2\right\}^{\left(\frac{l-\tau_1+l_1-1}{2}-2\right)} \left\{\left(k_2 + \lambda \frac{u'}{u} k_1\right)^2\right\}^{\left(\frac{l-\tau_1+l_1-1}{2}-2\right)}.$$

The final expression for M is obtained by substituting for $W_{\mu\nu}$ from Eq. (12) in Equation (3). The constant C in Eq. (12), however, is not determnied by conformal invariance. On physical grounds, we expect this constant C to be related to the pion decay constant. Equation (12) gives the general expression for an off-shell amplitude for the process $\pi^0 \to 2\gamma$. It is to be noted that the invariant amplitude is a function k_1^2, k_2^2 and $(k_1+k_2)^2$ as it should be. Further, the amplitude blows up for $l=2$ if twe take $\tau_1 = 2$ also.

The expression for $W_{\mu\nu}$ for large k_1^2, k_2^2 and P^2 takes the form

$$W_{\mu\nu} \xrightarrow[k_1^2,\, k_2^2,\, P^2 \text{ large}]{} \text{Constant} \times i\, e_{\mu\nu\alpha\beta}\, k_1^{\alpha}\, k_2^{\beta}\, (k_1^2)^{(l-\tau_1)}\, (k_2^2)^{(l-\tau_1)}$$
$$\times (k_1\cdot k_2)^{1-(l-\tau_1)} \int \ldots \int d\lambda\, d\lambda'\, du\, du'\; f_1(u)\, f_1(u')\, (u^2)^{(l-\tau_1-2)/2}\, (u'^2)^{(l-\tau_1-2)/2} \tag{13}$$
$$\times \{\lambda(1-\lambda)\}^{(l-\tau_1-2)/2}\, \{\lambda'(1-\lambda')\}^{(l-\tau_1-2)/2}\, \{(1-u)(1-u') - 4\}$$

where we have put $l_1 = 3$ (for a conserved vector field). The expression (13) shows that the power behaviour of the amplitude is determined by l, $2l$ being the physical (anomalous) dimension of the pion interpolating field which has been treated as a composite field. The physical dimension l imparts flexibility to the amplitude. The integrals appearing in the expression (13) can be computed for given values of l with the help of the relations (7a) and (7b) and making use of the Bateman manuscript project.

The asymptotic expression for $W_{\mu\nu}$ with $k_1{}^2 \to 0$, $k_2{}^2 \to 0$ and $P^2 \to 0$ is (with $l_1 = 3$)

$$W_{\mu\nu} \to \text{Constant} \times i\, e_{\mu\nu\alpha\beta}\, k_1{}^\alpha k_2{}^\beta\, (k_1 \cdot k_2)^{(l-\tau_1+1)} \tag{14}$$
$$\times \int \dots \int \mathrm{d}\lambda\, \mathrm{d}\lambda'\, \mathrm{d}u\, \mathrm{d}u'\;\; f_1(u)\, f_1(u')\, (u^2)^{(l-\tau_1-2)/2}\, (u'^2)^{(l-\tau_1-2)/2} \{\lambda(1-\lambda)\}^{(l-\tau_1-2)/2} \{\lambda'(1-\lambda')\}^{(l-\tau_1-2)/2}.$$

From the expression (14) one may be tempted to conclude that when $(k_1 \cdot k_2) \to 0$, $W_{\mu\nu} \to 0$. In fact, in considering the limiting value of $W_{\mu\nu}$ one has to recognise the importance of the power of $(k_1 \cdot k_2)$, namely, of $(l - \tau_1 + 1)$. In this limiting case only that value of l will contribute for which $l - \tau_1 + 1 = 0$. Obviously, $\lim_{(k_1 \cdot k_2) \to 0} (k_1 \cdot k_2)^{(l-\tau_1+1)} = 1$ with $l - \tau_1 + 1 = 0$. The non-vanishing of the invariant amplitude in the limit $k_1{}^2 \to 0$, $k_2{}^2 \to 0$ and $P^2 \to 0$ is indeed gratifying [4]. For the sake of simplicity, we have considered spin-zero and charge-zero partons as constituents of the pion. The method of derivation of the off-shell amplitude for the process investigated here can be generalized by considering spin-half charged partons relaxing also the number af partons which we have assumed to be two.

2. Conclusion

In Sect. 1 we have shown that the off-shell amplitude for the process $\pi^0 \to 2\gamma$ becomes asymptotically conformal invariant if the compositeness of the neutral pion is taken into considerations. There also we have derived, with the help of some simplifying assumptions, an expression for the off-shell amplitude for the process concerned employing parton model description of the neutral pion to take into account its compositeness. It is highly gratifying that the motivation of our investigation is nicely supported by the Johnson-Baker-Willey version of finite QED from which the asymptotic conformal invariance of the off-shell amplitude for the process $\pi^0 \to 2\gamma$ follows as a necessary consequence.

One remark is in order here. Our derivation of the off-shell amplitude for the process $\pi^0 \to 2\gamma$ is not rigorous because of our some simplifying assumptions. We have made an attempt in this paper to enlarge the scope of the parton model concept of the hadrons which has already been proved very useful in understanding some important aspects (such as, for example, scaling phenomena) of high energy physics. In fact, in this paper we have tried to show that the parton model gives us an alternative route to the pion physics in addition to the existing pion PCAC for some specific problems.

For the sake of simplicity we have considered the pion as a bound system of two non-interacting partons. However, a more realistic derivation of the off-shell amplitude for the process investigated here necessitates the consideration of the parton-gluon model to take into considerations the possible interactions amongst the partons the number of which may not necessarily be two as has been assumed in this investigation. To consider the interactions amongst the constituent partons in an algebraic approach employing conformal invariance one needs the chiral conformal group. The work in this direction will be very interesting as it will enlarge the domain of applications of the parton model and conformal symmetry as well.

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[1] J. Steinberger, Phys. Rev. **76**, 1180 [1949].
[2] J. S. Bell and R. Jackiw, Nuovo. Cim. **60 A**, 47 [1969].
[3] R. J. Crewther, Phys. Rev. Letters **28**, 1421 [1972].
[4] C. Nash, Nuovo. Cim. Letters **6**, 467 [1973].
[5] S. D. Drell, Phys. Rev. **D 7**, 2190 [1973].
[6] A. A. Migdal, Phys. Letters **37 B**, 98 [1971].
[7] R. P. Feynman, Phys. Rev. Letters **23**, 1415 [1969].
[8] J. D. Bjorken and E. A. Paschos, Phys. Rev. **185**, 1975 [1969].
[9] M. Ciafaloni and P. Menotti, Nuovo. Cim. Letters **6**, 545 [1973].
[10] K. Johnson, M. Baker, and R. Willey, Phys. Rev. **B 136**, 1111 [1964].
[11] K. Johnson and M. Baker, Phys. Rev. **D 8**, 1110 [1973].
[12] S. Ferrara, R. Gatto, and A. F. Grillo, Nucl. Phys. **B 34**, 349 [1971].
[13] K. Nishijima, Fields and Particles, Benjaman, New York 1969, p. 335.
[14] S. Ferrara, A. F. Grillo, and G. Parisi, Nuovo. Cim. **12 A**, 952 [1971].
[15] I. T. Todorov, Lecture Notes.